Quartile Deviation

An absolute measure of dispersion based on quartiles is called quartile deviation. It is also known as semi-inter quartile range.

Inter quartile range is the difference between the upper and lower quartile.

(ie) Inter quartile range $= Q_3 - Q_1$

Semi Inter quartile range

$$= \frac{Q_3 - Q_1}{2}$$

For an individual series,

$$Q_1 = \text{Value of } \left(\frac{n+1}{4}\right)^{th} \text{ item}$$

$$Q_3 = \text{Value of } \left[3\left(\frac{n+1}{4}\right)\right]^{th} \text{ item}$$

where n – number of items

For a discrete frequency distribution,

Q_1. is given by the formula

$$Q_1 = \text{Value of } \left(\frac{N+1}{4}\right)^{th} \text{ item}$$

where N – Total frequency

For a continuous frequency distribution,

$$Q_1 = L + \left(\frac{\frac{N}{4} - m}{f}\right) C$$

where L = Lower limit of the Q_1 – class

m = Cumulative frequency preceding the Q_1 – class

f = Frequency of the Q_1 – Class

$$Q_3 = L_1 - \left(\frac{\frac{3N}{4} - m_1}{f_1} \right) C$$

and

when L_1 – Lower limit of the Q_3 class

m_1 – Cumulative frequency preceding the Q_3 class

f_1 – Frequency of the Q_3 class

C – Length of the class interval

Illustration: [Individual series]

Find the upper and lower quartiles of the following data. Also find the quartile deviation.

36, 43, 47, 28, 18, 9, 32

Solution:

Arranging the values in order we have

9, 18, 28, 32, 36, 43, 47

$n = 7$

$$\therefore Q_1 = \text{Value of } \left(\frac{n+1}{4} \right)^{th} \text{ item}$$

$$= \text{Value of } \left(\frac{7+1}{4} \right)^{th} \text{ item}$$

$$= \text{Value of } 2^{nd} \text{ item}$$

$$Q_1 = 18$$

$$Q_3 = \text{Value of } 3\left(\frac{n+1}{4} \right)^{th} \text{ item}$$

$$= \text{Value of } 3\left(\frac{7+1}{4} \right)^{th} \text{ item}$$

$$= \text{Value of } 6^{th} \text{ item}$$

$$Q_3 = 43$$

$$\therefore Q.D = \frac{Q_3 - Q_1}{2}$$

$$= \frac{43 - 18}{2} = 12.5$$

Exercise

Find the Inter quartile range and quartile deviation of the following data.

1. Amount of Bill paid by 10 patients in a hospital are

    Rupees: 5700, 6800, 7900, 3100, 9200, 10300, 1400, 12500, 4600, 3600.

2. Number of accidents taking place in 10 main cities of India over a particular year are

    630, 720, 510, 400, 395, 277, 138, 349, 565, 365.

3. A survey was carried out in hospitals to find the percentage of people admitted in.

    15, 26, 38, 40, 24, 37, 10, 6

4. Amount of ten project groups allotted with a stipend amount per month are

    12500, 11675, 10750, 9500, 8000, 10500, 5750, 6500, 7725, 8930

Illustration: [Discrete Frequency distribution]

Calculate quartile deviation of the following data of marks obtained by 50 students.

Marks :	10	20	30	40	50	60
No. of Students :	4	7	15	8	7	9

Solution

Marks	Frequency	Cumulative Frequency
10	4	4
20	7	11
30	15	26·
40	8	34
50	7	41
60	7	50 ·
	50	

N = Total Frequency

= 50

$$\therefore \quad Q_1 = \text{Value of } \left(\frac{N+1}{4}\right)^{th} \text{ item}$$

$$= \text{Value of } \left(\frac{50+1}{4}\right)^{th} \text{ item}$$

$$= \text{Value of } 12.75$$

$$= \text{Value of } 26 \text{ (next highest value of 12.75 in the Cf column)}$$

$$Q_1 = 30$$

$$Q_3 = \text{Value of } 3\left(\frac{N+1}{4}\right)^{th} \text{ item}$$
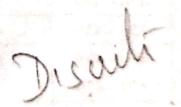
$$= \text{Value of } 3(12.75)$$

$$= \text{Value of } 38.25$$

$$= \text{Value of } 41 \text{ [Next highest value of 38.25 in the cf column]}$$

$$Q_3 = 50$$

$$\therefore \quad Q.D = \frac{Q_3 - Q_1}{2}$$

$$= \frac{50 - 30}{2}$$

$$= \frac{20}{2} = 10$$

Discuti

Exercise

Calculate the Q.D of the following

1.

Vitamin In take/day (mgs) :	15	25	35	45	55	65	75
No. of families :	32	42	104	110	83	36	14

2. Multiplication factors for Conversion to substance concentration in plasma

Substance :	1	2	3	4	5	6	7	8	9	10
Multiplication factor :	0.784	0.587	0.250	0.5	0.157	1.179	0.411	0.323	0.166	0.714

3. The amount of yield of rice from 10 acre paddy field were found to be

Field :	1	2	3	4	5	6	7	8	9	10
Yield (kgs) :	1880	1760	1640	1600	1920	1800	1860	1960	1750	1850

4. The plasma protein fractions after separation are isolated in varying concentrations:

Ptn :	1	2	3	4	5	6	7
(%) :	52	65	13	8	22	29	54

Illustration:

Calculate the semi-Inter quartile range of the following data:

Weight (gms)	410 – 419	420 – 429	430 – 439	440 – 449	450 – 459	460 – 469	470 – 479
No of apples	14	20	42	54	45	18	7

Solution:

Weight (gms)	No.of apples	Corrected class interval	Cumulative Frequency	
410 – 419	14	409.5 – 419.5	14	
420 – 429	20	419.5 – 429.5	34	Q_1 class
430 – 439	42	429.5 – 439.5·	76	
440 – 449	54	439.5 – 449.5	130	Q_3 class
450 – 459	45	449.5 – 459.5	175	
460 – 469	18	459.5 – 469.5	193	
470 – 479	7	469.5 – 479.5	200	
	200			

$\Sigma f = N$ = Total Frequency

= 200

For $Q_1 = \dfrac{N}{4} = 50$; $C = 10$

L = 429.5 ; f = 42 , m = 34

$$\therefore Q_1 = L + \left(\frac{\frac{N}{4} - m}{f} \right) C$$

$$= 429.5 + \left(\frac{50 - 34}{42} \right) 10 = 433.3$$

$$\boxed{Q_1 = 433.3}$$

For Q_3 :

$$\frac{3N}{4} = \frac{3(200)}{4} = 150;$$

$$C = 10$$
$$L = 449.5$$
$$f = 45$$
$$m = 130$$

$$\therefore Q_3 = L + \left(\frac{\frac{3N}{4} - m}{f} \right) C$$

$$= 449.5 + \left(\frac{150 - 130}{45} \right) 10$$

$$Q_3 = 453.9$$

$\therefore$ Semi – Inter quartile range

$$\frac{Q_3 - Q_1}{2}$$

$$= \frac{453.9 - 433.3}{2}$$

$$= 10.322$$

Coefficient of Quartile Deviation

$$\frac{Q_3 - Q_1}{Q_3 + Q_1}$$

$$= \frac{453.9 - 433.3}{453.9 + 433.3} = 0.0116$$

Exercise

Calculate the Quartile deviation and its relative measures of dispersion of the following data:

1.

Calcium Intake / day in (g):	0.5 – 0.6	0.6 – 0.7	0.7 – 0.8	0.8 – 0.9	0.9 – 1.0	1.0 – 1.1	1.1 – 1.2
No of Children:	45	48	27	29	32	35	30

2.

Weight of rats(g):	91 – 100	101 – 110	111 – 120	121 – 130	131 – 140	141 – 150
No.of. rats:	30	36	24	25	20	15

3.

Weight (g):	410 – 420	420 – 430	430 – 440	440 – 450	450 – 460	460 – 470
No.of Oranges :	14	12	3	5	1	2

Remark

(i) For a symmetrical distribution Q_3 – Median = Median – Q_1

$$\text{(ie) Median} = \frac{Q_3 + Q_1}{2} = Q_1 + Q.D$$

$$= Q_3 - Q.D$$

(ii) For a skewed distribution Median $\neq Q_1$ + Quartile deviation $\neq Q_3$ – Quartile deviation.

Mean Deviation:

Range and Inter quartile range depends only on the two values, extreme values. Since the above two measures are not computed, involving all the items of the given series, it is considered to be inferior.

To compute the measure of dispersion based on all the items of the given series, we have a measure called mean deviation.

Definition

Mean deviation is the arithmetic mean of the absolute deviations of various items from Mean, Median (or) Mode.

Steps to be followed to compute the Mean deviation

(i) Compute the average [Mean (or) Median (or) Mode] of a given data
(ii) Calculate the deviation d = x – average where x – is the value of the item
(iii) Find $|d|$, [this is obtained by ignoring (+) (or) – sign of the deviation.

Apply the formula

(a) $\text{M.D} = \frac{\Sigma |d|}{n}$ [Individual series] where n = no. of items

(b) $M.D = \dfrac{\sum f|d|}{n}$ [For a frequency distribution] where N = Total Frequency

Note

$|d|$ – is read as "Modulus d"

Relative Measure of M.D

Coefficient of Mean deviation is known as the relative measure of mean deviation.

It is the ratio between the mean deviation and the average [Mean (or) Mode (or) Median] chosen for deviation.

(i.e.,) Coefficient of mean deviation = $\dfrac{M.D}{\text{Mean (or) Median (or) Mode}}$

Illustration; (individual series)

Compute the mean deviation for the following weights (g) of 6 fishes 54, 48, 43, 47, 62, 40.

Solution

Since the given series is an individual series.

$$A.M = \bar{X} = \frac{\sum x}{n}$$

$$\frac{295}{6} = 49.17 = 49 \text{ (approx)}$$

$$\therefore d = x - \bar{x} = x - 49$$

x	$d = x - \bar{x}$	$|d|$
54	$54 - 49 = 5$	5
48	$48 - 49 = -1$	1
43	$43 - 49 = -6$	6
47	$47 - 49 = -2$	2
62	$62 - 49 = 13$	13
41	$41 - 49 = -8$	8
295		35

$$\therefore \text{ Mean deviation} = \frac{\Sigma|d|}{n}$$

$$= \frac{36}{6} = 5.833$$

$$\text{Coefficient of mean deviation} = \frac{M.D}{A.M}$$

$$= \frac{5.833}{49} = 0.1190$$

Exercise

Calculate the mean deviation of the following

1. Height (cm) of 7 children are

   10, 12, 56, 90, 92, 94, 93

2. Marks of 6 students are

   32, 42, 72, 82, 62, 52, 92

3. Number of trees in 8 different places are

   3, 9, 15, 24, 11, 6, 8, 17

4. Number of patients in 8 hospitals located at various places are

   52, 58, 64, 76, 78, 84, 90, 43

5. Weight of 7 frogs are (gms)

   24, 28, 40, 36, 48, 32,, 44

Illustration: [Frequency distribution]

Compute the mean deviation for the following weights (g) of 40 fishes

Weight (g)	No.of Fishes
10	5
20	2
30	10
70	10
80	12
90	1
	40

Solution

x	f	fx	$d = x - \bar{x}$	$\|d\|$	$f\|d\|$
10	5	50	− 43.5	43.5	217.5
20	2	40	− 33.5	33.5	67
30	10	300	− 23.5	23.5	235
70	10	700	16.5	16.5	165
80	12	960	26.5	26.5	318
90	1	90	36.5	36.5	365
	40	2140		180	1039

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = 53.5 = \frac{2140}{40} = 53.5$$

$$\therefore \text{Mean deviation} = \frac{\Sigma f|d|}{\Sigma f}$$

$$= \frac{1039}{40}$$

$$= 25.975$$

$$\therefore \text{Coefficient of M.D} = \frac{M.D}{\bar{x}}$$

$$= \frac{25.975}{53.5} = 0.486$$

Note: In case of a continuous series, x is the mid value of the class intervals.